

Characterization of high finesse mirrors: loss, phase shifts and mode structure in an optical cavity

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An extensive characterization of high finesse optical cavities used in cavity QED experiments is described. Different techniques in the measurement of the loss and phase shifts associated with the mirror coatings are discussed and their agreement shown. Issues of cavity field mode structure supported by the dielectric coatings are related to our effort to achieve the strongest possible coupling between an atom and the cavity.

I. INTRODUCTION

For many contemporary physics experiments, measurement enhancement via an optical cavity is a useful tool. Indeed, an optical cavity allows one to extend the interaction length between matter and field, to build up the optical power, to maintain a well-defined mode structure, and to study the extreme nonlinear optics and quantum mechanics associated with the large field of a single photon for small cavity volumes [1]. In most situations, a better understanding of cavity and mirror properties is important for achieving improved sensitivity and for elimination of systematic errors. For example, in cavity QED, one needs to know the mode structure of the intracavity field in order to develop the optimum strategy of atom-cavity coupling; for frequency metrology, accurate determination of phase shifts of the resonant fields can provide precision frequency markers; and in quantitative spectroscopy, knowledge of the mirror loss sets the accuracy scale of absorption measurement. On the technology development side, the knowledge gained from careful mirror characterization could provide guidelines for the optic coating community to develop in situ measurement and control capabilities of the coating process.

The work presented in this paper is motivated by the ever-increasing demand for a high coherent coupling rate between an atom and the field, as well as of a decreasing cavity loss rate. The aim is to have coherent (reversible) evolution dominating over dissipative processes, and thereby to explore manifestly quantum dynamics in real time, which in turn should lead eventually to the investigation of the strong conditioning of system evolution on measurement results and the realization of quantum feedback control. An important feature associated with strong coupling is that system dynamics are readily influenced by single quanta. Thus single-atom and single-photon cavity QED provides an ideal stage where the dynamical processes of individual quantum systems can be isolated and manipulated. A collection of such coherent systems could help to realize a distributed quantum networks for computation and communication [2]. At each node the quantum information is stored by one

or a collection of entangled atoms. Photons serve as the communication link which in turn entangle the whole network. Within this context, technical advances in optical cavity quantum electrodynamics have become increasingly important. Some significant developments down this road have been achieved by the group at Caltech [3, 4, 5, 6, 7, 8], where in Ref. [8] the one-photon Rabi frequency is $\Omega_1/2\pi = 220\text{MHz}$, in comparison with the atomic decay rate $\gamma_\perp = 2.6\text{MHz}$ and the cavity decay rate $\kappa/2\pi = 14.2\text{MHz}$.

The strong coupling condition $\Omega_1 \gg (\gamma_\perp, \kappa)$ is achieved by using a small cavity length, of the order of $10\mu\text{m}$. Precise measurement of the length of a short optical cavity facilitates the determination of mirror coating characteristics. A $10\mu\text{m}$ cavity length translates to a free-spectral-range (*FSR*) of 15 THz, or a wavelength difference of a few tens of nanometers (for example, it is 36 nm for a center wavelength of 852 nm) for neighboring cavity modes. Therefore a straightforward 6-digit measurement of the wavelengths (Burleigh wavemeter) of the cavity modes acquires a precision of the order of 5×10^{-5} for accurate determination of the equivalent optical length of the cavity, from which details of the index of refraction and layer thickness of materials in the mirror stack can be inferred.

The low loss rate of the cavity field is made possible by high quality mirror coatings that lead to scatter and absorption losses in the 10^{-6} range [9, 10]. The cavity finesse and overall cavity transmission can be measured directly to determine the mirror losses, l , and transmission, T . This information can be combined with the *FSR* measurement in two useful ways: Firstly, the *FSR* measurement is sensitive to the difference in refractive index $n_H - n_L$ of the materials making up the multilayer mirror stack, whereas the transmission T depends on the ratio n_H/n_L , as will be shown later. As a result, a precise measurement of both the *FSR* and T can be used to determine the values of n_H and n_L independently. Moreover, by mapping out the wavelength dependence of the *FSR*, the thickness of layers in the mirror stack can be determined. Secondly, if one of the refractive indices (here n_L) is well known, then the *FSR* measurement determines n_H , and an independent value for the

mirror transmission T can then be calculated from n_H and n_L , and compared to the experimentally measured result. Indeed, the work presented in this paper shows that we are able to make complementary and mutually-confirming measurements of the cavity properties by the two approaches, i.e., measurements of the direct cavity loss and the dispersion of the cavity modes.

Coming back to the cavity QED experiments, we note that knowledge of the cavity properties is of importance in two particular ways: 1. Mirror absorption/scatter losses are a critical limiting factor in the loss rate from our cavity QED system - for our current cavities the loss rate from photon scattering due to mirror imperfections is similar in size to the atomic spontaneous emission rate. To build robust quantum computing/communications devices from cavity QED components, it is necessary to improve the ratio of mirror transmission to mirror losses. 2. The standing-wave light field inside the cavity penetrates into the mirror coatings, giving a larger mode-volume V_{mode} than would be expected naively from the physical distance between the mirror surfaces. Since $\Omega_1 \propto 1/\sqrt{V_{\text{mode}}}$, as our micro-cavities are pushed to shorter lengths, this leakage field will have a non-negligible effect on the achievable coupling strength $g_0 = \Omega_1/2$.

II. DIRECT TRANSMISSION AND LOSS MEASUREMENTS

All of the mirrors described in this paper were fabricated by Research Electro-Optics in Boulder, Colorado[10]. More specifically, the measurements were made for the particular coating run REO #T95 and involved mirrors with radius of curvature $R = 10$ and 20 cm. The coating run had a design transmission of $T^{\text{th}}=7\text{ppm}$ at a center wavelength of 852nm , from which a cavity finesse of $\mathcal{F}=370,000$ was expected. It was somewhat surprising therefore to measure a finesse of $\mathcal{F}=480,000$ at the targeted wavelength, and this prompted us to make more detailed measurements of the mirror properties, and design a model to match these measurements.

Firstly, losses were measured directly with a $40\mu\text{m}$ length cavity of 20cm radius of curvature mirrors in the usual way by recording resonant cavity transmission, reflection and finesse. If we denote the transmission of mirrors 1 and 2 by T_1 and T_2 , respectively, and the (absorption + scatter) loss per mirror as $l_i = (A + S)_i$, then the total cavity losses $\mathcal{L} = T_1 + T_2 + l_1 + l_2$ can be determined from the cavity finesse $\mathcal{F} = FSR/2\kappa$, with FSR as the cavity free spectral range and κ as the HWHM for the TEM_{00} mode of the cavity; equivalently, $\mathcal{F} = 2\pi/\mathcal{L}$. The cavity linewidth $\beta = 2\kappa$ can be determined from a ring-down measurement or using a modulation sideband as a frequency marker with the cavity length scanned, which is the technique employed here. The cavity transmission $I_{\text{trans}} = \frac{4T_1T_2}{(T_1+T_2+l_1+l_2)^2}$ can then be used to determine

$l_1 + l_2$, if T_1 and T_2 are known independently. In practice this is a difficult measurement to make, because the overall transmission I_{trans} depends on the mode-matching into the cavity being perfect. A variation of this protocol that does not require perfect mode-matching can be derived, by comparing the cavity reflection and transmission values with the cavity locked on resonance and off resonance.

The rudiments of this protocol are as follows. First of all, the total loss ($\mathcal{L} = T_1 + T_2 + l_1 + l_2$) is always measured first with the determination of the cavity FSR and linewidth. Now let us denote the input power as P_{in} , the reflected power P_r , and the transmitted power P_t . There is also a mode matching factor ϵ , meaning that of the input power of P_{in} , only ϵP_{in} is useful for coupling to the cavity TEM_{00} mode, $(1 - \epsilon)P_{in}$ is wasted. We have the following equations (the assumption of two equal mirrors is reasonable since the two mirrors are produced in the same coating run)

$$\mathcal{F} = \frac{2\pi}{T_1 + T_2 + l_1 + l_2} = \frac{\pi}{l + T} \quad (1)$$

$$\frac{P_t}{\epsilon P_{in}} = 4T_1T_2\left(\frac{\mathcal{F}}{2\pi}\right)^2 = T^2\left(\frac{\mathcal{F}}{\pi}\right)^2 \quad (2)$$

$$\frac{P_r - (1 - \epsilon)P_{in}}{\epsilon P_{in}} = (l_1 + l_2 + T_1 - T_2)^2\left(\frac{\mathcal{F}}{2\pi}\right)^2 = l^2\left(\frac{\mathcal{F}}{\pi}\right)^2 \quad (3)$$

Remember that $(1 - \epsilon)P_{in}$ is the “useless” power that is reflected directly off of the input mirror, and must be subtracted from P_r to leave the reflected power we wish to measure, that is, the sum of the field leaked from the cavity storage and the field (mode-matched) directly reflected off the input mirror. This cavity contrast is a direct result of the mirror properties. Division of equation 2 by 3 gives

$$\frac{P_t}{P_r - P_{in}} = \frac{T^2\left(\frac{\mathcal{F}}{\pi}\right)^2}{l^2\left(\frac{\mathcal{F}}{\pi}\right)^2 - 1} \quad (4)$$

Equation 4, combined with 1, will determine completely T and l .

In the actual experiment, this direct measurement approach found that (from finesse we have $l + T = 7.2\text{ppm}$) $P_{in} = 54\mu\text{W}$, $P_r = 42.6\mu\text{W}$ and $P_t = 4.82\mu\text{W}$ and therefore $l = 2.9\text{ppm}$ and $T = 4.3\text{ppm}$, with measurement uncertainties below 5%.

Another way to measure the (T, l) is by sweeping out all the high order spatial modes and carefully noting the transmission and reflection powers at each spatial mode. One measures the total input power and also sums together the powers of every matched mode for transmission and reflection. These three powers can be used in

Eq. 2 and 3 to calculate the partition between T and l . That measurement produced $l = 3ppm$, and $T = 4.2ppm$. The value of T should be a bit lower in this case because it is not possible to include all higher order modes in the measurement, some of them are simply impossible to resolve due to their weakness.

Other cavities measured with mirrors from the same coating run had higher finesse, very likely due to a lower density of surface defects. To construct a cavity of minimal mode volume for the intended maximal coherent coupling rate, we need to have the distance between two mirrors (radius of curvature $R = 10cm$) on the order of $10\mu m$ or below. To avoid contact between the outer edges of the two mirrors, the mirrors were fabricated with cone-shaped fronts, reducing the substrate radius from 3 mm to 1 mm. We notice this extra machine process might have introduced some additional surface defects on some mirrors. However, the highest finesse achieved with cone-shaped mirrors was comparable to unmodified pieces, at $\mathcal{F} = 480,000 \pm 10,000$, corresponding to losses $l = 2.2ppm$ if mirror transmission $T = 4.3ppm$ as determined from the above measurements.

III. TECHNICAL DETAILS OF THE MODEL

In this section we derive a model for the coating properties. A transfer-matrix formalism was used to calculate the input-output propagation of a plane-wave field through the 37 layer stack of alternating high index (Ta_2O_5 , $n_H = 2.0411$) and low index (SiO_2 , $n_L = 1.455$) dielectric layers (these dielectric constants are assumed to be constant with wavelength). The substrate refractive index (supplied by REO) used was $n_{sub} = 1.5098$. That is, the transfer of the field through each $\lambda/4$ layer is represented by a matrix, and the response of the entire mirror (or cavity) is determined by the product of these individual matrices.

Following the treatment of Hecht [11] for normal incidence, we take the matrix representing layer j to be given by $M_j = \begin{bmatrix} \cos(kh_j) & (i \sin(kh_j))/Y_j \\ iY_j \sin(kh_j) & \cos(kh_j) \end{bmatrix}$. Here M_j relates the electric and magnetic fields (E, H) of the input and output via

$$\begin{bmatrix} E_{out} \\ H_{out} \end{bmatrix} = [M] \begin{bmatrix} E_{in} \\ H_{in} \end{bmatrix}. \quad (5)$$

$k = 2\pi/\lambda$ is the free-space wavevector of the incident light, $h_j = n_j \times (\text{layer thickness})$ with n_j the refractive index, and $Y_j = \sqrt{\frac{\epsilon_0}{\mu_0}} n_j$ with (ϵ_0, μ_0) the electric and magnetic constants in SI units. For an exact $\lambda/4$ layer (and for light at the design wavelength of the coating), this simplifies to $M_j = \begin{bmatrix} 0 & i/Y_j \\ iY_j & 0 \end{bmatrix}$. A multi-layer stack is represented by multiplying the matrices of the individual layers: For light incident on layer 1,

the matrix for the entire structure of q layers is defined as the product $M = M_1 M_2 \dots M_q$. For our mirror stack, this gives $M = (M_{Ta_2O_5} M_{SiO_2})^{18} M_{Ta_2O_5}$. Note that at the coating center (where there is an exact $\lambda/4$ layer),

$$M_{Ta_2O_5} M_{SiO_2} = \begin{bmatrix} -\frac{n_L}{n_H} & 0 \\ 0 & -\frac{n_H}{n_L} \end{bmatrix}, \text{ so the system matrix}$$

$$\text{has the simple form } M = \begin{bmatrix} 0 & \frac{i}{Y_H} \left(\frac{n_L}{n_H}\right)^{18} \\ iY_H \left(\frac{n_H}{n_L}\right)^{18} & 0 \end{bmatrix}.$$

For a field incident from material with index n_0 and exiting into material with index n_s , the resulting transmission coefficient is given by

$$t = 2Y_0 / (Y_0 M_{11} + Y_0 Y_S M_{12} + M_{21} + Y_S M_{22}), \quad (6)$$

with transmission $T = \frac{n_s}{n_0} |t|^2$ (the factor $\frac{n_s}{n_0}$ accounts for the change in the amplitude of the electric field in the dielectric, thereby conserving the net energy flux). At the center wavelength of the coating then,

$$T = \frac{n_s}{n_0} \left| -2i / [(n_S/n_H)(n_L/n_H)^{18} + (n_H/n_0)(n_H/n_L)^{18}] \right|^2. \quad (7)$$

We can make a further simplification: as $(n_L/n_H)^{18} = 0.0018$ and $(n_H/n_L)^{18} = 557$, the first term in the denominator of the above equation is only a 10^{-6} correction, so the final result for T at the coating center becomes

$$T = 4n_S n_0 (n_L)^{36} / (n_H)^{38}, \quad (8)$$

and the transmission is determined by the *ratio* of the refractive indices.

This calculation reproduced the target reflectivity of $T^{th} = 7.3ppm$ for the coating run #T95, and $T^{th} = 14.6ppm$ for another REO coating run #D1306 where the number of layers was reduced to 35. The model and measured (REO spectrophotometer data) “coating curves” are shown in Fig. 1 for the #D1306 coating run.

For a fixed cavity length the resonance wavelengths of the cavity can be calculated simply with the same transfer-matrix formalism, using a matrix for the entire system, $M_{total} = M M_{gap} M$, (a product of two mirrors plus a fixed-length vacuum gap in between). The calculation steps through a series of wavelengths calculating the cavity transmission T at each, and by finding places of maximum transmission finds the vacuum wavelengths of the cavity resonances.

Conversely, for a given set of measured cavity-resonance wavelengths, it is possible to determine the effective cavity length precisely. With a commercial wavelength-meter that gives 6-digit wavelength measurement, we typically measure the cavity resonance within an uncertainty of 0.01 nm. Error propagation analysis gives an uncertainty for the determination of the effective cavity length (tens of microns) on the order of 0.05 – 0.1 nm. The parameters of the model (index contrast, layer

thickness) are set by comparison to such measurements. Hence, armed with the detailed knowledge of the mirrors provided from the model, the physical cavity length can be determined precisely from a single measurement of resonance, for example, when the cavity is locked to a laser of known frequency (in our case a cesium transition at 852.359nm). Close to the center of the design wavelength of the coating, the effective cavity length (on resonance) is roughly $L_{eff} = L + 1.633\lambda/2$ with L (the physical distance between the mirror surfaces) an integer number of $\lambda/2$. The physical cavity length can therefore be determined, with an uncertainty of ~ 0.5 nm, limited by the overall parameter-fitting in the model. Further details of the wavelength-dependence are provided by reference to the model.

IV. FREE-SPECTRAL RANGE (FSR) MEASUREMENTS

To determine the parameters of the model (index contrast, layer thickness), a series of precise measurements of the cavity FSR (frequency between successive cavity resonances) was made[12, 13, 14]. At fixed cavity length a Ti-Sapphire laser was tuned to find successive resonant wavelengths (λ_1, λ_2) of the cavity, and an experimentally determined length was then defined by

$$L_{expt} = \lambda_1 \lambda_2 / 2(\lambda_1 - \lambda_2).$$

This length comprises the actual physical length between the two mirror surfaces, L , plus a contribution from leakage of the mode into the mirror stack, which gives rise to an additional phase shift at the coatings, to give a length $L_{eff} > L$. In addition, the leakage into the coatings increases with wavelength as (λ_1, λ_2) move away from the coating design wavelength, so this gives another additional contribution to the round-trip phase and hence to the measured length L_{expt} .

As discussed in Ref. [12], if λ_1 and λ_2 were closely spaced compared to the scale on which the coating properties vary (so that coating dispersion could be neglected) then near the design wavelength of the coating, we would have $L_{expt} = L_{eff} = L + (\frac{1}{n_H - n_L}) \times \lambda_c / 2$ where n_H and n_L are the high and low index materials of the stack, and $\lambda_c = 2\lambda_1 \lambda_2 / (\lambda_1 + \lambda_2)$ is the average (in frequency) of wavelengths λ_1 and λ_2 . We thereby have a dependence of the free spectral range on $(\frac{1}{n_H - n_L})$, which combined with the transmission (which depends on n_L/n_H) can fix n_H and n_L . For these materials, this gives $L_{eff} = L + 1.633\lambda_c/2$. However, for our measurements with short cavities, λ_1 and λ_2 are separated by ~ 30 nm, so $L_{expt} > L_{eff}$. But we can still use the complete model to fit to the measured values (λ_1, λ_2) and determine parameters of the coating. Finally, by mapping out this wavelength dependence of the free-spectral range to find $\min(L_{expt})$, we find the center wavelength of the coating.

In the model, the refractive indices used are adjusted to obtain the same pairs (λ_1, λ_2) as measured. Then, the layer thickness in the model is adjusted to agree with the measured coating center wavelength. By using the additional information of the measured mirror transmission T from Section II, we can now either:

1. Derive independent values for the refractive indices and layer thickness, or
2. Assuming one index is known, use the refractive indices and layer thickness information to give an independent value for the mirror transmission, which can be compared to the measurement of Section II.

That the dispersion (FSR) measurement alone is sufficient to determine the loss-less part of the mirror properties represents some useful information for the mirror coating technician: the index difference $n_H - n_L$ and the optical thickness of the coating layers can be simply measured in this way without interference from absorption/scatter losses. And, if n_L is known, this also gives a simple way of finding the mirror transmission. Adding in a direct measurement of mirror transmission yields values for n_H and n_L separately.

Data obtained from these measurements are shown in Figure 2, where L_{expt} is plotted as a function of wavelength, for a $10 \mu\text{m}$ cavity with 10 cm radius of curvature mirrors. The circles are measured data, and the curves the calculation from the model, with parameters chosen to best fit the data. This data was taken by setting the cavity to a series of different lengths, and recording a pair

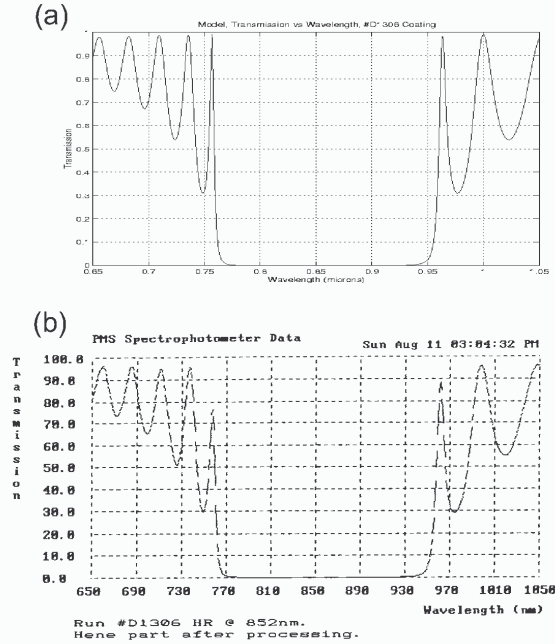


FIG. 1: (a) Calculated and (b) Measured transmission of coating as a function of wavelength, for a 35 layer $\lambda/4$ stack with $n_H=2.0411$, $n_L=1.455$, and center wavelength 850nm.

of resonant wavelengths (λ_1, λ_2) at each length. The x axis is center wavelength $\lambda_c = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$, the y axis the measured cavity length $L_{expt} = \lambda_1\lambda_2/2(\lambda_1 - \lambda_2)$ shown in units of $\lambda_1/2$: for each pair (λ_1, λ_2), the length is such that $L_{expt}/(\lambda_1/2) = 24.xx$. Dividing by λ_2 instead would exactly give 23.xx. Since by rearranging the formula for L_{expt} we see that $L_{expt}/(\lambda_1/2) \equiv L_{expt}/(\lambda_2/2) + 1$. Due to a finite drift in the cavity length, each measurement of λ was made to only 5 digits resolution (e.g. 852.59 ± 0.01 nm), leading to the uncertainty in L_{expt} shown. Uncertainty in λ_c is ± 0.03 nm and cannot be seen on this scale.

Two theory curves are shown. The solid curve shows a model with n_L assumed to be fixed at its nominal value of $n_L = 1.455$. To best fit the data, n_H was *increased* to $n_H = 2.0676$ (a factor of 1.3%). In addition, the center wavelength was shifted to 847 nm (by reducing the thickness of each $\lambda/4$ layer by 0.6%). Discussions with REO confirmed that 1.3% is a known offset in n_H for the particular coating machine which produced this run, and also that a few nm uncertainty in the center wavelength is typical. With these parameters, the inferred mirror transmission is $T_{inf} = 4.6 \pm 0.2$ ppm, agreeing well with the measured value $T_{exp} = 4.3$ ppm from Section II. The dotted curve (which overlaps the solid curve) shows the model when both n_L and n_H are allowed to vary. Their values are chosen to match both the FSR measurement shown, and to give a mirror transmission to match exactly the experimentally determined value $T_{exp} = 4.3$ ppm. Parameters which satisfy these criteria are $n_H = 2.0564$ (0.75% increase) and $n_L = 1.4440$ (0.76% decrease). Our direct measurement of T in Section II had a large uncertainty, which limits the absolute determination of n_H and n_L to about this 1% level. However, a more precise measurement could in principle determine the indices at the 0.1% level. One application might be to measure T and the FSR as a function of position across a mirror substrate, thereby mapping out stress induced variations in the refractive indices at the 0.1% level with a spatial resolution of ~ 10 μ m.

In this data set the correction for the Gaussian phase difference between the actual resonator mode and the plane-wave of the model has been neglected. After the propagation distance from the mode waist to the mirror surfaces, a Gaussian beam will have acquired less phase than a plane wave traveling the same distance. For a 10 μ m cavity with 10 cm radius of curvature mirrors, this gives a 2% correction, corresponding to a shift in L_{expt} by $\simeq 0.0045$ cavity orders (that is, $\Delta L \simeq +\frac{\lambda}{2} \times 0.0045$). Lowering the refractive index contrast of the model to shift the calculated curve by this amount would increase the inferred mirror transmission by $\lesssim 0.1$ ppm. For our second cavity (44 μ m, 20 cm radius of curvature mirrors), the correction is 0.0066 cavity orders.

The mirror phase shift (FSR measurement) is only sensitive to the transmission (index contrast) and center wavelength (layer thickness). Therefore, if absorp-

tion/scatter losses are added to the model (by introducing an imaginary component to the refractive index) the cavity resonance wavelengths do not change. More precisely, adding a scattering loss at the mirror surfaces has exactly zero effect on the FSR and mirror transmission. Adding losses *within* the coatings has a small effect: increasing the mirror absorption from 0.5 ppm to 2 ppm (an experimentally reasonable range) changes the mirror transmission by a factor of $\simeq 10^{-5}T$, clearly negligible, and again there is no effect on the FSR measurement. As a result, this measurement (with n_L assumed fixed) provided a very simple and sensitive inference of the mirror transmission of $T_{inf} = 4.6 \pm 0.2$ ppm, which is *unaffected* by absorption/scatter losses.

The same measurement and fitting procedure was used on another cavity with mirrors from the same coating run. This 44 μ m cavity made from 20 cm radius of curvature mirrors gave a transmission of $T_{inf} = 4.5 \pm 0.2$ ppm, with a center wavelength of 848 nm. (This was the cavity used for the direct measurements of Section II which gave $T = 4.3$ ppm).

One other factor which has been ignored so far is the effect of fluctuations in the $\lambda/4$ layer thickness. Discussions with REO suggested that a 1% variation in thickness was reasonable, so a Gaussian-distributed variation (of standard deviation 1%) was added to the layer thicknesses of the model. For cavity calculations, identical mirrors were used for both sides of the cavity. The principal effect of this variation is to shift the center wavelength of the coating - over several realizations of random coatings, this resulted in an rms shift of the center wavelength by ± 1.2 nm. So, the measured shift of center wavelength in the coating (from 852 nm to 847 nm) is probably due partly to a systematic offset, and partly to fluctuations. The mirror transmission is also affected: the value of the

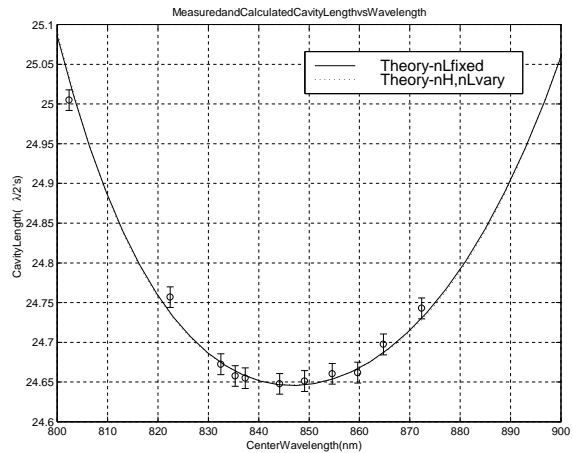


FIG. 2: The cavity length L_{expt} measured from the Free-Spectral Range (FSR) varies about the design wavelength of the coating. Fitting a model to these data points gives a measure of mirror transmission (from fitting of the difference $n_H - n_L$) and center wavelength (from fitting layer thickness).

transmission is on average *increased* slightly, by 0.6% in the case studied, from 4.55 ppm to 4.58 ppm at the center of the coating. At the level of our current measurements this is another negligible effect, but with a more precise measurement aimed at determining n_H and n_L , the possibility of a systematic offset from this mechanism should be considered. Lastly, the FSR measurement is mostly effected via the change in center wavelength of the coating - the value of $\min(L_{\text{expt}}(\text{simulated}))$ has a mean the same as without the added fluctuations, and varies by only 0.0014 mode orders rms, again negligible for our purposes.

Another useful result of these calculations is that the free spectral range of the cavity is well known, so that resonant wavelengths of the cavity can be accurately predicted. This is important for choosing a diode laser of correct center wavelength to match the mode, for applications such as cavity locking or dipole-force traps. With the idea of using a laser of >920 nm wavelength to form an intracavity dipole-force trap,[15] this knowledge was particularly important: our Ti:Sapphire laser tuned only as high as 890nm so cavity resonances in this wavelength range could not be measured, only predicted. With the parameters chosen above for the model, the following theoretical and experimental resonance wavelengths resulted:

theory	787.208nm	818.659	853.255	890.798	930.683
experiment	787.170nm	818.651	853.255	890.800	N/A

The experimental value for the cavity resonance can then confidently be predicted to be 930.7 ± 0.05 nm, and a diode laser chosen accordingly.

V. LIMITATIONS TO MODE VOLUME

In a similar calculation to the one described above, it is possible to calculate the field distribution of light inside the resonant cavity, by describing each layer separately with a left and right travelling plane wave, then matching electromagnetic boundary conditions between layers. An example of this kind of calculation is shown in Figure 3, where refractive index and field distribution (modulus of the electric-field) are plotted as a function of distance for a cavity with length $L_{\text{eff}} = 3\lambda/2$. The coupling strength g_0 of an atom placed in the center of the cavity mode is proportional to $\frac{1}{\sqrt{V_m}}$, where V_m the cavity mode volume is found by integrating the field ($D \cdot E$) over the standing wave and Gaussian transverse mode profile. Large coupling is achieved by making a short cavity with a small mode waist (short radius of curvature mirrors).

For a cavity of physical length L , the “leakage” of the mode into the $\lambda/4$ mirror stack (look at the tails of the mode in Fig. 3) that increases L to L_{eff} also increases the cavity mode volume. For our materials at 852nm, $L_{\text{eff}} = L + 1.633\lambda/2$, so for a cavity with physical distance between mirror surfaces $L = \lambda/2$, the cavity mode volume ends up being 2.63 times larger than might otherwise have been expected, and hence the atom-cavity

coupling g_0 is 0.6 times smaller than the naive estimate based on the physical separation of the mirror surfaces.

This effect is proportionately larger as the cavity length gets shorter. In Figure 4, the expected g_0 is plotted for a cavity formed with two 20cm radius of curvature mirrors, as a function of the physical distance L between the mirrors. The two curves show a real mirror (with g_0 reduced by leakage into the coatings) and an idealized mirror with no leakage (perfect reflectors at $\pm L/2$). The transverse (Gaussian waist) dimension is calculated by simple Gaussian beam propagation, which is not strictly accurate for length scales less than a few microns; however any error in this should be roughly the same for both the ideal and actual mirror cases, so the ratio of these should remain sensibly correct. The cavity is assumed resonant at an integer number of half-wavelengths of light at the 852nm Cs D2 transition; that is, each $\lambda/2$ is a distance of 0.426 microns.

The discrepancy between the expected and achieved coupling g is large even for our longer cavities - 5% for a $10\mu\text{m}$ cavity. However, in the lab this is largely compensated by the fact that we never measure the actual physical distance L between mirror surfaces, but instead $L_{\text{expt}} = \lambda_1\lambda_2/2(\lambda_1 - \lambda_2)$, which is close to L_{eff} , and so incorporates the same offset of mirror penetration that determines g_0 . This method of length measurement breaks down eventually due to the dispersion of the mirror coatings: Eventually if λ_1 is at the center of the coating, λ_2 will be so far separated in wavelength that it reaches the edges of the mirror coating stopband, and the observed round-trip phase has then more to do with the structure of the dielectric coatings than it does with the vacuum gap between the surfaces of the cavity mirrors. That is to say, our measured L_{expt} becomes increasingly different from L_{eff} and introduces an offset in estimating the mode volume as the cavity length approaches the scale of the wavelength.

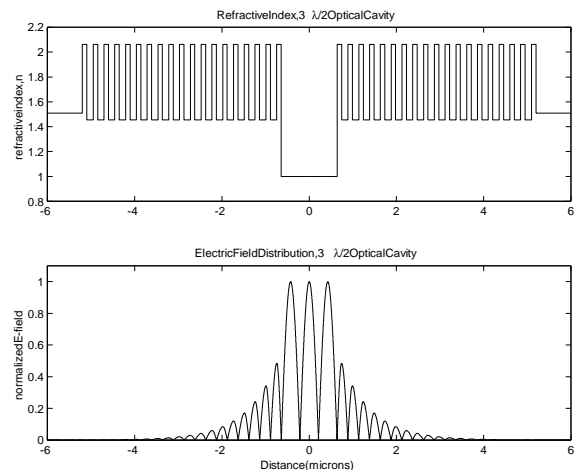


FIG. 3: Mirror refractive index stack design, and resulting electric field distribution for a resonant $3\lambda/2$ cavity made from dielectric mirrors.

At $L = 20\lambda/2$ physical length (the regime of our present cavities) the difference between the coupling coefficient g_0 inferred from L_{expt} and that found by integrating $D \cdot E$ over the mode volume is $<0.1\%$. At $L = 10\lambda/2$ ($4.26\mu\text{m}$) it would be a 1% error, at $5\lambda/2$ an 8% error. Note however that knowledge of these offsets means that when calculating g_0 from L_{expt} we can compensate for this effect. Measurements of L_{expt} for cavities any shorter than $5\lambda/2$ would be impossible since λ_2 has reached the edge of the mirror stopband. To align shorter cavities a new method for length measurement will need to be developed, such as measuring the frequency spacing of transverse modes.

We are now in a position to estimate parameters for the best Fabry-Perot cavity that will be experimentally feasible in the near future using this type of mirrors. First consider a $L = \lambda/2$ cavity with 20cm radius of curvature mirrors. If the mirror transmission and losses were each reduced to $T = l = 0.5\text{ppm}$ to yield a cavity finesse of $\mathcal{F} = 3.14 \times 10^6$, then this cavity has parameters $(g_0, \kappa, \gamma_{\perp})/2\pi = (647, 56, 2.6)\text{MHz}$, which gives critical photon number $n_0 = \gamma_{\perp}^2/2g_0^2 = 8.1 \times 10^{-6}$ and critical atom number $N_0 = 2\kappa\gamma_{\perp}/g_0^2 = 7.0 \times 10^{-4}$. To make a cavity of this length the 20cm mirrors would have to be reduced to a diameter of 0.5mm rather than 1mm. At this size there would still be a $0.11\mu\text{m}$ gap between the mirror edges for the $L = \lambda/2$ ($0.426\mu\text{m}$) cavity length, which should make it possible to still get atoms into and out of the cavity (as in Ref. [3, 4, 5, 6, 7, 8]), and to align the mirrors.

If the mirror diameter could be reduced to $350\mu\text{m}$ (without adversely affecting the cavity losses), then 10cm radius of curvature mirrors could be used, with a $0.12\mu\text{m}$ gap at the edges. Due to the tighter radius

of curvature, $g_0/2\pi$ would be increased to 770MHz in this case. Now speculating that “dream” mirrors of $T = 0.2\text{ppm}$ transmission, $l = 0.2\text{ppm}$ loss might be possible ($\mathcal{F} = 7.85 \times 10^6$), we could aim for the ultimate goal of $(g_0, \kappa, \gamma_{\perp})/2\pi = (770, 22, 2.6)\text{MHz}$, in which case $n_0 = 5.7 \times 10^{-6}$ photons, and $N_0 = 1.9 \times 10^{-4}$ atoms.

In conclusion, we have presented two measurement approaches, one based upon direct loss and the other on cavity dispersion, that produce the same quantitative determination of the mirror coating properties. The dispersion measurement is more informative, as it has the potential to determine the complete characteristics of a mirror. A model has been derived to link the mirror properties to the physical parameters of coating layers. Issues relevant to optical cavity QED, such as the cavity field mode structure, have been discussed.

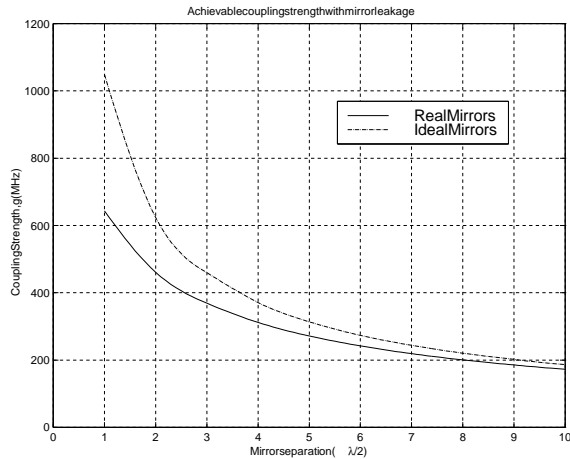


FIG. 4: Coupling coefficient g_0 (expressed in cycles per second) versus the physical separation L of the surfaces of two mirrors forming a Fabry-Perot resonator. Due to penetration of the standing-wave mode into the mirror coatings, the cavity mode volume achieved with real mirrors is larger (and hence the coupling strength smaller) than for an ideal mirror with the same spacing between mirror surfaces but no penetration.

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